Poloidal Flow Driven by Ion-Temperature-Gradient Turbulence in Tokamaks

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We show that linear collisionless processes do not damp poloidal flows driven by ion-temperature-gradient (ITG) turbulence. Since these flows play an important role in saturating the level of the turbulence, this level, as well as the transport caused by ITG modes, may be overestimated by gyrofluid simulations, which employ linear collisionless rotation damping. [S0031-9007(97)05109-0]

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Recent advances in gyrofluid simulation of ion-temperature-gradient (ITG) modes in tokamaks have shown that the predominant saturation mechanism for the instability is the production of axisymmetric, primarily poloidal flows [1–3] which vary with radius and serve to shear stabilize the instability. The damping of such poloidal flows is, thus, critically important in determining the turbulence level to be expected. Less damping will result in smaller thermal transport rates. The adequacy of gyrofluid models for calculating the damping is an important issue, especially in view of "first principles" claims that such ITG turbulence would severely limit confinement in reactor-sized tokamaks [4,5]. In this Letter, we solve kinetically for the linear collisionless damping of poloidal flows, treating the ITG turbulence drive as a known source, and also as an initial value problem.

Our result, in contrast to gyrofluid predictions, is that these flows, even if driven by a rapidly fluctuating source, are not damped by collisionless processes (Landau damping). The gyrofluid equations were derived from the gyrokinetic equation by taking moments and closing the moment hierarchy by approximations which model kinetic effects. These include linear damping terms which are correct for the nonaxisymmetric ITG modes, but are incorrect for the axisymmetric poloidal flows.

We use the gyrokinetic description of the plasma [6] to determine its response to a source. The distribution functions for electrons and ions are given by $f = F_0 + \delta f$, where F_0 is the equilibrium, which we choose to be Maxwellian at temperature T, and which is assumed to vary slowly perpendicular to a magnetic surface. The perturbed part of the distribution function is written as $\delta f = -(e\phi/T)F_0 + g$, where ϕ is the potential and, to lowest order in the gyrokinetic expansion, $g = g(\vec{R}, \varepsilon, \mu, t)$. The guiding center position is $\vec{R} \equiv \vec{x} - \vec{\rho}$, where $\vec{\rho} = \hat{b} \times \vec{v}/\Omega$ is the gyroradius, and $\Omega = eB/(mc)$ is the gyrofrequency. The independent velocity variables used are the energy $\varepsilon = v^2/2$ and the magnetic moment $\mu = v_\perp^2/(2B)$. The rapid spatial variation perpendicular to the magnetic field is assumed to be contained in an eikonal function: $\phi(\vec{x}) = \phi_k \exp[iS(\vec{x}_\perp)]$, and similarly for $g(\vec{R})$. The wave vector is defined by $\vec{k}_\perp = \nabla S$.

We consider only an axisymmetric source (i.e., $n = 0$), since the response to nonaxisymmetric sources would clearly be Landau damped. Then the eikonal is a function of ψ only: $S = S(\psi)$, where ψ is the poloidal flux function. The gyrokinetic equation in this case is

$$\frac{\partial g_k}{\partial t} + v_\parallel \hat{b} \cdot \nabla g_k + i\omega_\mathrm{D} g_k = \frac{e}{T} F_0 J_0 \frac{\partial \phi_k}{\partial t} + S_k F_0 , \tag{1}$$

where $J_0 = J_0(k_\perp \rho)$ is a Bessel function. The drift frequency is defined by $\omega_\mathrm{D} = \vec{k}_\perp \cdot \vec{v}_\mathrm{d}$, where $\vec{v}_\mathrm{d}$ is the guiding center drift velocity: $\vec{v}_\mathrm{d} = -v_\parallel \hat{b} \times \nabla(v_\parallel/\Omega)$, where $v_\parallel = [2(\varepsilon - \mu B)]^{1/2}$. For axisymmetric perturbations,

$$\omega_\mathrm{D} = (\vec{v}_\mathrm{d} \cdot \nabla\psi) S'(\psi) = K v_\parallel \hat{b} \cdot \nabla(v_\parallel/B) , \tag{2}$$

where $K = (mcI/e)S'(\psi)$, with $I \equiv RB_\phi$. The potential is determined by the quasineutrality condition

$$-\frac{e}{T_\mathrm{i}} n_0 \phi_k + \int d^3v\, J_0 g_{ik} = \frac{e}{T_\mathrm{e}} n_0 \phi_k + \int d^3v\, g_{ek} . \tag{3}$$

The source causes charge to build up on magnetic surfaces because of finite ion gyroradius and banana orbit width.

The source S_k represents the $\vec{E} \times \vec{B}$ nonlinearity in the gyrokinetic equation [6], which is due to the nonzero n modes of the ITG turbulence. It does not depend on the $n = 0$ modes for which we solve. Although some low n modes may be weakly damped and may have effects on the high n modes similar to the $n = 0$ modes, we have considered only the $n = 0$ modes because they are strictly undamped.

We consider the long-time evolution of the rotation driven by a fixed steady source. Then, with the appropriate expansion, the zeroth order (in the bounce time) equation is

$$v_\parallel \hat{b} \cdot \nabla g_0 + i\omega_\mathrm{D} g_0 = 0 , \tag{4}$$

whose solution has the form $g_0 = h \exp(-iKv_\parallel/B)$, where $\hat{b} \cdot \nabla h = 0$. The first order equation is

$$v_\parallel \hat{b} \cdot \nabla g_1 + i\omega_\mathrm{D} g_1 = -\frac{\partial g_0}{\partial t} + \frac{e}{T} F_0 J_0 \frac{\partial \phi_k}{\partial t} + S_k F_0 , \tag{5}$$

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which yields the solubility condition determining h:

$$\frac{\partial h}{\partial t} = \frac{e}{T} F_0 \overline{\left(e^{iQ} J_0 \frac{\partial \phi_k}{\partial t} \right)} + \overline{(e^{iQ} S_k)} F_0 , \qquad (6)$$

where $Q = K v_\parallel / B$. The bounce average is defined by $\overline{A} = \oint (dl/v_\parallel) A / \oint (dl/v_\parallel)$, where $dl = B dl_{\rm p}/B_{\rm p}$; for trapped particles, the integral goes over a closed orbit, while for untrapped particles, it goes once around the poloidal circumference. Thus, for times longer than a few bounce times, g_k is given to lowest order by

$$g_k = e^{-iQ} \left[\frac{e}{T} \overline{(e^{iQ} J_0 \phi_k)} + \overline{(e^{iQ} R_k)} \right] F_0 , \qquad (7)$$

where $R_k = \int dt\, S_k$. The finite banana width effects are contained in the e^{iQ} factors. The electron distribution function is given by setting $J_0 = 1$ and $Q = 0$.

Quasineutrality then yields the integral equation for ϕ_k:

$$n_0 e \left(\frac{1}{T_{\rm i}} + \frac{1}{T_{\rm e}} \right) \phi_k - \frac{e}{T_i} \int d^3 v\, F_{0i} e^{-iQ} J_0 \overline{(e^{iQ} J_0 \phi_k)} - \frac{e}{T_{\rm e}} \int d^3 v\, F_{0e} \overline{\phi_k} = s_k , \qquad (8)$$

where the source terms are combined in the expression

$$s_k = \int d^3 v\, F_{0i} e^{-iQ} J_0 \overline{(e^{iQ} R_{ik})} - \int d^3 v\, F_{0e} \overline{R_{ek}} . \qquad (9)$$

Note that, for a time-independent source, s_k and ϕ_k grow linearly in time.

The integral equation can be solved with the use of a variational principle. Writing Eq. (8) as $\mathcal{L}\, \phi_k = s_k$ and defining an inner product as the integral over a magnetic surface $(u, v) = \oint (dl_{\rm p}/B_{\rm p}) uv$, the variational expression can be written as

$$V = \frac{(\phi_k^*, \mathcal{L}\, \phi_k)}{|(\phi_k^*, s_k)|^2} . \qquad (10)$$

Using $d^3 v = 2\pi d\varepsilon B d\mu / |v_\parallel|$, the numerator can be written as

$$(\phi_k^*, \mathcal{L}\, \phi_k) = e \sum_j \frac{1}{T_j} \int 2\pi\, d\varepsilon\, d\mu\, F_{0j} \times \left[\oint \frac{dl}{v_\parallel} |\phi_k|^2 - \frac{|\oint (dl/v_\parallel) e^{iQ} J_0 \phi_k|^2}{\oint (dl/v_\parallel)} \right] . \qquad (11)$$

It is straightforward to show that V is minimized for the exact solution of the integral equation, and that the minimum value is $V = 1/(\phi_k^*, s_k) = 1/(\phi_k^*, \mathcal{L}\, \phi_k)$. Equation (11) can be shown to be positive definite, using the Schwartz inequality and $1 - J_0^2 \geqslant 0$. Because a positive minimum of V exists, it follows that a nontrivial solution of the integral equation exists. Therefore, the linear potential response to the axisymmetric part of the source increases with time without saturating, i.e., it is not damped by collisionless processes, but only by much weaker collisional effects, not included here.

In deriving this result, we have assumed that the time scales of interest are much longer than a bounce time for a thermal ion. We will later justify this assumption: See Eqs. (16)–(18). Although resonances and collisionless damping do occur on a shorter time scale, the axisymmetric potentials survive for longer times, being modified only by plasma polarization. It is true that there is a class of very low energy particles with small bounce frequencies, which could provide a resonance, but these are few in number and their effect is neglected. Including the g_1 correction from Eq. (5) would give only a small correction to our results.

In order to obtain more specific results, we must be more specific about the source. We assume that the electrons are adiabatic and do not contribute to the source because they cannot move across magnetic surfaces. The ion source S_{ik} must be of order $k_\perp^2 \rho^2$ for small gyroradius. We expand in powers of the ion gyroradius and the ion banana width, using $e^{iQ} J_0 \simeq 1 + iK v_\parallel / B - (K v_\parallel / B)^2/2 - (k_\perp \rho)^2/4$. To lowest order, we have

$$(\phi_k^*, \mathcal{L}_0 \phi_k) = e \sum_j \frac{1}{T_j} \oint \frac{dl_{\rm p}}{B_{\rm p}} \int d^3 v\, F_{0j} |\phi_k - \overline{\phi_k}|^2 = 0 , \qquad (12)$$

which implies that ϕ_k must be uniform on a magnetic surface: $\hat{b} \cdot \nabla \phi_k = 0$. Its value is determined by the next order equation: $(\phi_k^*, \mathcal{L}_1 \phi_k) = (\phi_k^*, s_k)$, or

$$\frac{e}{T_{\rm i}} \phi_k = \frac{1}{\mathcal{D}} \oint \frac{dl_{\rm p}}{B_{\rm p}} \int d^3 v\, F_{0i} \times \{ R_{\rm even} + iK [v_\parallel / B - \overline{(v_\parallel / B)}] R_{\rm odd} \} , \qquad (13)$$

where we have written the source in terms of even and odd parts (in $v_\parallel$): $\int dt\, S_{ik} = R_{\rm even} + R_{\rm odd}$. Here,

$$\mathcal{D} = \oint \frac{dl_{\rm p}}{B_{\rm p}} \int d^3 v \times F_{0i} \{ K^2 [\overline{(v_\parallel / B)^2} - \overline{(v_\parallel / B)}^2] + \overline{(k_\perp^2 \rho^2)}/2 \} \qquad (14)$$

represents the shielding effects of a collisionless neoclassical polarization current, as well as the classical polarization current.

As a specific example, we consider a source which is independent of poloidal angle, i.e., the $m = 0$ part; taking S_{ik} to be independent of velocity, and using large aspect ratio circular geometry, these integrals can be expressed as elliptic integrals and evaluated explicitly [7], with the result

$$\frac{e \phi_k}{T_{\rm i}} = (1 + 1.6 q^2 / \epsilon^{1/2})^{-1} \int dt\, S_{ik} / (k_\perp^2 a_{\rm i}^2) , \qquad (15)$$

where $a_{\rm i}^2 = (T_{\rm i}/m_{\rm i})/\Omega_{\rm i}^2$, $\epsilon = r/R$ is the inverse aspect ratio, and $q = \epsilon B / B_{\rm p}$ is the tokamak safety factor. For small ϵ and $q > 1$, the shielding is dominated by the neoclassical polarization. We have assumed that the plasma is deep in the banana regime, so that the ion-ion collision frequency is small enough that collisional corrections to the neoclassical polarization are not important.

Although the source may be rapidly varying in time, the long time response which we have determined is what is needed to show that the mean square potential increases with time in a way which is inconsistent with long-term linear collisionless damping. The linear response to the source can be written generally as

$$\phi_k(t) = \int_0^t dt'\, \mathcal{K}(t - t') S_k(t'). \tag{16}$$

The ensemble average of $|\phi_k|^2$ (related to the shear decorrelation of the ITG turbulence) is

$$\langle|\phi_k|^2\rangle = \int_0^t dt' \int_0^t dt''\, \mathcal{K}^*(t - t')\mathcal{K}(t - t'') \times \langle S_k^*(t') S_k(t'')\rangle. \tag{17}$$

Assuming the source is random and statistically stationary, $\langle S_k^*(t') S_k(t'')\rangle$ is a function of $|t' - t''|$ only; we assume it is nonzero for $|t' - t''| \lesssim \tau_c$ only, where τ_c is the autocorrelation time of the source. We are interested only in times $t \gg \tau_c$, so

$$\langle|\phi_k|^2\rangle \simeq 2\tau_c \langle|S_k|^2\rangle \int_0^t dt' |\mathcal{K}(t')|^2 \simeq 2\tau_c\langle|S_k|^2\rangle |\mathcal{K}|^2 t, \tag{18}$$

using our result $\mathcal{K} = \text{const}$ for times longer than a few bounce times. The mean square potential fluctuation increases linearly with time, neglecting collisions and nonlinear turbulent viscosity. This is inconsistent with linear collisionless damping, contained in the gyrofluid models. This may partly explain the discrepancy between the transport predictions of gyrofluid [1] and gyrokinetic [8] codes, but that is a complicated issue which is still being investigated, and we do not attempt a full explanation.

A model equation for the evolution of poloidal rotation would be

$$\frac{\partial}{\partial t}|\phi_k|^2 = A_k|S|^2 - B_k|\phi_k|^2 - C_k|\phi_k|^2|S|^2 - D_k\nu_{ii}|\phi_k|^2, \tag{19}$$

where $|S|^2$ represents the turbulent nonaxisymmetric fluctuations, and the coefficients A_k through D_k depend, in detail, on the wavelength spectrum and the nature of the ITG turbulence. We have calculated A_k and shown that B_k should vanish (although it apparently does not in gyrofluid models).

A rough criterion for saturating the turbulence is that the flow velocity shear determined by $|\phi_k|^2$ exceeds the linear growth rate of the modes. This saturation level, of course, determines the thermal diffusivity. Thus, a key issue, as yet unresolved, is whether the turbulent viscosity C_k is so strong as to make our linear damping calculation irrelevant (and also whether gyrofluid calculations of the viscosity are correct). Ultimately, this can probably be decided only by comparisons, now underway, between gyrofluid [9] and gyrokinetic [10,11] codes. If the C_k term is small, as near marginal stability, then $|\phi_k|^2$ will be inversely proportional to ν_{ii}, indicating improved confinement in larger, hotter tokamaks.

For comparison with linear gyrofluid and gyrokinetic code results, we consider the related problem of the collisionless relaxation of an initially poloidally rotating plasma. The solution of this problem can be obtained by using the source $S_{ik} = \delta f_k(0)\delta(t)$, where $\delta f_k(0)$ is the initial perturbed distribution function. The delta function in time is to be interpreted as a function whose width is much larger than a gyroperiod but much smaller than a bounce time. As a simple example, we choose the initial conditions to correspond to ion density and parallel flow perturbations:

$$\delta f_k(0) = \left[\frac{\delta n_k(0)}{n_0} + \frac{m_\mathrm{i}}{T_\mathrm{i}} v_\parallel u_{\parallel k}(0)\right] F_{i0}. \tag{20}$$

Then, identifying $R_\mathrm{even} = \delta n_k(0)/n_0$, $R_\mathrm{odd} = m_\mathrm{i} v_\parallel \times u_{\parallel k}(0)/T_\mathrm{i}$, the long time potential is given by Eq. (13). The initial ion density perturbation is accompanied by a potential perturbation because of quasineutrality and the classical polarization current: $\rho_\mathrm{pol}(0) + e\delta n_k(0) = 0$, where $\rho_\mathrm{pol}(0) = -(n_0 e/\Omega_\mathrm{i}) k_\perp^2 (c/B)\phi_k(0)$. This initial shielding occurs before the neoclassical polarization is established. We take the initial parallel flow to be of the form $u_{\parallel k}(0) = \alpha B(1 + \lambda\cos\theta)$, where α and λ are constants and θ is the poloidal angle. The integrals in Eq. (13) can be carried out with the result

$$\frac{e\phi_k}{T_\mathrm{i}} = (1 + 1.6q^2/\epsilon^{1/2})^{-1} \times \left[\frac{e\phi_k(0)}{T_\mathrm{i}} + (1.6\epsilon^{3/2} + 0.8\lambda\epsilon) \times \left(\frac{ik_\perp \alpha B/\Omega_{ip}}{k_\perp^2 a_\mathrm{i}^2}\right)\right], \tag{21}$$

where $\Omega_{ip} = eB_\mathrm{p}/(m_\mathrm{i} c)$.

The initial $E \times B$ and parallel velocities combine to give initial poloidal and toroidal velocities, which are related by $ik_\perp\phi_k = (u_\mathrm{p} B_\phi - u_\phi B_\mathrm{p})/c$ and $u_{\parallel k} = (u_\mathrm{p} B_\mathrm{p} + u_\phi B_\phi)/B$. The contribution to the perpendicular flow from the pressure gradient perturbation is smaller by a factor of $k_\perp^2 a_\mathrm{i}^2$ than that from the potential perturbation, and is neglected. This flow can be directly compared with the fluid flows determined from solving the gyrofluid equations. Using toroidal momentum conservation $u_\phi = u_\phi(0)$, and specializing to the case $\lambda = 0$, the final poloidal velocity can be expressed in terms of the initial poloidal and toroidal velocities:

$$u_\mathrm{p} = (1 + 1.6q^2/\epsilon^{1/2})^{-1} u_\mathrm{p}(0), \tag{22}$$

independent of $u_\phi(0)$. This result has been verified by a gyrokinetic simulation [10], while attempts to study linear rotation damping with gyrofluid codes [9] show strong anomalous linear damping. We further note that, due to the bounce averages occurring in Eq. (13) (the trapped particle effects), coupling of $m \neq 0$ sources to $m = 0$ modes is generally stronger than in fluid theories.

In conclusion, we have shown that the $n = 0$ poloidal flows driven by ITG turbulence, although modified by plasma polarization, are not linearly damped by collisionless processes. At least near marginal stability, where nonlinear damping of the poloidal flows should be negligible, and in the sufficiently collisionless regimes of interest (deep in the banana regime), the level of poloidal rotation should be larger, and the ITG turbulence level and transport should be considerably smaller than predictions made by gyrofluid simulations which entail linear collisionless damping. We note that the gyrofluid codes could be improved by using closures consistent with our results.

Since recent experimental results on core enhanced confinement seem to be explained by $\vec{E} \times \vec{B}$ flow shear suppression of turbulence [12], our results make enhanced confinement in large tokamaks appear more likely.

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[1] M. A. Beer, Ph.D. thesis, Princeton University, 1995.
[2] G. W. Hammett *et al.*, Plasma Phys. Controlled Fusion **35**, 973 (1993).
[3] R. E. Waltz, G. D. Kerbel, and J. Milovich, Phys. Plasmas **1**, 2229 (1994).
[4] M. Kotschenreuther *et al.*, in Proceedings of the 16th International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Montréal, 1996 (International Atomic Energy Agency Report No. IAEA-CN-64/D1-5, 1996), (International Atomic Energy Agency, Vienna, to be published).
[5] J. Glanz, Science **274**, 1600 (1996).
[6] E. A. Frieman and L. Chen, Phys. Fluids **25**, 502 (1982).
[7] M. N. Rosenbluth and F. L. Hinton, Nucl. Fusion **36**, 55 (1996).
[8] A. M. Dimits *et al.*, Phys. Rev. Lett. **77**, 71 (1996).
[9] G. W. Hammett and M. A. Beer (private communication).
[10] A. M. Dimits (private communication).
[11] R. D. Sydora (private communication).
[12] K. H. Burrell, Phys. Plasmas **4**, 1499 (1997).